# Evolution of flat universe with a cosmological term in modified Relativistic Theory of Gravitation as a scalar-tensor extension of General Relativity

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**Abstract**

We consider the dynamics of tensor and scalar gravitational fields in the Relativistic Theory of Gravitation with the Minkowskian vacuum metric and generalize the formulation to the massless graviton. The potential of scalar field is determined in the presence of cosmological term under clear physical motivations. We find cosmological inflationary solutions and analyze conditions providing the transition to the regime of hot expanding universe.

PACS numbers: 04.50.+h, 98.80.Cq, 03.50.-z

# I. Introduction

Recently the astronomical observations on Supernovas with high red shifts [1, 2] shown that the most probable value for the fraction of cosmological term in the density of energy differs from zero even in the case of flat Universe (see Fig. 1 from [1]). Such the measurements result in the average values of parameters determining the fractions of energy densities for the matter and cosmological contribution in ratios to the critical density of energy for the flat Universe, so that $\Omega_M = 0.28^{+0.09+0.05}_{-0.08-0.04}$ and $\Omega_\Lambda \approx 0.72$, respectively, with the same error bars.

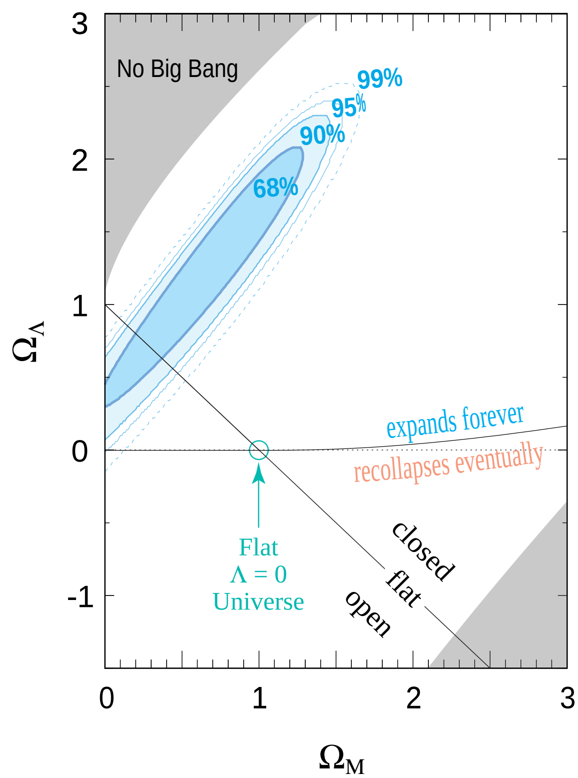


Figure 1: Results of [1] represented in the plane of $\Omega_M$ and $\Omega_\Lambda$ parameters. The flat universe corresponds to the line $\Omega_\Lambda + \Omega_M = 1$.

For the nonrelativistic motion of matter the deceleration parameter of universe expansion is determined by the expression

$$q = \frac{\Omega_M}{2} - \Omega_\Lambda,$$

which leads to the estimate [1, 2, 3]

$$q = -0.33 \pm 0.17. \tag{1}$$

Thus, the experimental data point to the actuality and necessity of comprehensive analysis on the evolution of flat universe in the presence of cosmological term.

The cosmological constant in the flat universe is a necessary ingredient in the Relativistic Theory of Gravitation (RTG) [4]. The construction of RTG by A.A.Logunov was based on the fundamental physical principle of vacuum stability, so that, according to RTG, the vacuum is

the flat Minkowskian space-time. In this way the metric tensor of Riemannian space-time is considered as the Faraday–Maxwell field over the flat vacuum. The field equations of motion have to satisfy the following principles:

1. The vacuum stability: the gravitation field equations are identically true under the absence of matter fields, so that the metric tensor of Riemannian space-time is equal to the Minkowskian metric.

2. The conservation law: the divergence of energy-momentum tensor for the gravitational field under the covariant derivative composed by the connection consistent with the Minkowskian metric is identically equal to zero.

3. The weak equivalence principle (the geometrization): the interaction of gravitational field with the matter is built under the Riemannian metric.

The theory of gravitational field constructed in [4] leads to the Lagrangian, which coincides with the Einstein–Hilbert Lagrangian in the theory of General Relativity, if only the cosmological constant is equal to zero. A nonzero cosmological term implies the introduction of additional contribution, which is related with the mass of graviton in RTG. This mass can be introduced in the presence of background, vacuum metric, the Minkowskian one in the case under study. In [4] the theory with the degenerated masses of tensor graviton (spin 2) and scalar gravitational field (spin 0) was constructed. The additional fields of spins 1 and 0 are eliminated from RTG due to the consequence from the conservation law (the second principle), which makes the divergence of Riemannian metric under the covariant Minkowskian derivative be equal to zero. Thus, the RTG is a bi-metric theory reducible to a scalar-tensor variant for the extension[1] of standard Lagrangian for the gravitational field in the General Relativity, but the ambiguity in the choice of functional parameters in the theory is completely cancelled due to the clear physical restrictions given above.

The motivation of RTG in the light of modern view to the problem is not restricted by the principle to construct the theory for the Faraday–Maxwell field of gravitation. Indeed, the geometrization principle for the interaction of gravity with the matter leads to the nonrenormalizability of quantum theory as in the General Relativity. However, we can suggest that we deal with the classical low-energy field theory under virtualities less than the Planck scale, while a full renormalizable theory is formulated in the flat space-time[2], and the dimensional Planck scale appears under the breaking of conformal invariance in the renormalization group and the spontaneous breaking of higher symmetry in the full theory [6]. In this way the stable vacuum in the form of Minkowskian metric can enter the effective low-energy Lagrangian for the gravitational field, that takes place in RTG in the case of nonzero cosmological constant. On the other hand, if we part with the principle of renormalizability, then we can suppose that a full theory is not local, i.e. it deals with some extended objects, that necessarily leads to an extension of full high-energy space-time dimension [7]. These additional dimensions have to be compactified at the energies below the Planck scale, so that the vacuum is determined by a configuration, which can enter the effective classic theory of gravity. In the case of flat four-dimensional vacuum, the Lagrangian can depend on the Minkowskian metric, that again

[1]Some examples of bi-metric theories of gravity and scalar-tensor variants are presented in [5].
[2]In the curved space-time the local quantum field theory is certainly nonrenormalizable.

points to the actuality of consideration with the theory-construction principles accepted in RTG.

In [4] the gravitational field equations are derived in the case of degenerate masses of tensor and scalar fields. In the study of universe evolution in that version of RTG, authors found stringent restrictions on the graviton mass, that followed from the age of Universe. The cosmological solution in the presence of matter was found pulsating with the non-inflationary expansion, which has lots of problems [8]. Moreover, the deceleration parameter occurred greater than $\frac{1}{2}$, that contradicts the mentioned astronomical observations. In general, the rather strict suggestion on the connection of cosmological constant with the nonzero masses of gravitational fields can be avoided. In the present paper we modify the RTG by studying massless graviton and graviscalar in the presence of nonzero cosmological term. The exploration of RTG principles leads to a motivated determination of potential for the scalar field, which equation of motion can be exactly integrated out. In this way the dimensional parameter giving the cosmological constant is not connected to the masses of gravitational fields. The offered modification of RTG results in significant cosmological implications. Namely, the evolution of universe at the initial stage has the inflationary solution (see a comprehensive description of inflationary scenario in [8]). The value of cosmological constant falls under the inflation, so that at late times under the transition to flat expanding universe the deceleration parameter agrees with the recent astronomical observations. We show at which conditions the equations of motion in the cosmology lead to the regime of hot expanding universe.

In Section II we construct the dynamics of modified RTG (MRTG). Then we find the solutions for the evolution of universe in the presence of cosmological constant in MRTG with the dust or ultra-relativistic matter in Section III. The obtained results are summarized and discussed in Conclusion.

## II. Dynamics of tensor and scalar gravitational fields

According to RTG the density of Riemannian metric $\mathfrak{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}$ is expressed in terms of sum of densities for the Minkowskian metric and gravitational Faraday–Maxwell field [9]

$$\begin{aligned} \mathfrak{g}^{\mu\nu} &= \sqrt{-\gamma}\,(\gamma^{\mu\nu} + \Phi^{\mu\nu}), \\ D_\mu\, \Phi^{\mu\nu} &= 0, \end{aligned} \tag{2}$$

where $D_\mu$ is the covariant derivative with the Christoffel symbols determined by the Minkowskian metric, so that they globally become zero in the Galilean (Cartesian) coordinates in the whole space-time, while the zero divergence actually is the implication of conservation law for the energy-momentum tensor, that will be shown later. The gravitational field can be expanded to the traceless tensor part and the scalar contribution, so that

$$\begin{aligned} \Phi^{\mu\nu} &= \left[\Phi^{\mu\nu} - \tfrac{1}{3}\left(\gamma^{\mu\nu} - \tfrac{D^\mu D^\nu}{D^2}\right)\left(\gamma_{\alpha\beta}\Phi^{\alpha\beta}\right)\right] + \tfrac{1}{3}\left(\gamma^{\mu\nu} - \tfrac{D^\mu D^\nu}{D^2}\right)\omega' \\ &= u^{\mu\nu} + w'^{\mu\nu}, \\ D_\mu\, u^{\mu\nu} &= 0, \quad \gamma_{\mu\nu}\, u^{\mu\nu} = 0, \\ \mathfrak{u}^{\mu\nu} &= \sqrt{-\gamma}\, u^{\mu\nu} = \sqrt{-\gamma}\, \left[\Phi^{\mu\nu} - \tfrac{1}{3}\left(\gamma^{\mu\nu} - \tfrac{D^\mu D^\nu}{D^2}\right)\left(\gamma_{\alpha\beta}\Phi^{\alpha\beta}\right)\right], \\ \mathfrak{w}'^{\mu\nu} &= \sqrt{-\gamma}\, w'^{\mu\nu} = \sqrt{-\gamma}\, \tfrac{1}{3}\left(\gamma^{\mu\nu} - \tfrac{D^\mu D^\nu}{D^2}\right)\omega'. \end{aligned} \tag{3}$$

We introduce an independent density caused by the scalar field in the form

$$\mathfrak{w}^{\mu\nu} = \mathfrak{w}'^{\mu\nu} + \sqrt{-\gamma}\,\gamma^{\mu\nu} = \sqrt{-\gamma}\,\frac{1}{3}\left(\gamma^{\mu\nu} - \frac{D^\mu D^\nu}{D^2}\right)\omega, \tag{4}$$

where $\omega = 4 + \omega'$, and at zero $\omega'$ we have got the flat metric[3]. It is significant that the contraction of Riemannian and Minkowskian metric tensors depends on the scalar component of gravitational field, only, so that

$$\gamma_{\mu\nu}\mathfrak{g}^{\mu\nu} = \gamma_{\mu\nu}\mathfrak{w}^{\mu\nu},$$

where, in accordance with our definitions, we put

$$\mathfrak{g}^{\mu\nu} = \mathfrak{u}^{\mu\nu} + \mathfrak{w}^{\mu\nu}.$$

The density of Lagrangian for the gravitational field in the modified RTG with the cosmological constant is expressed in the form[4]

$$\mathfrak{L} = -\frac{1}{4}\,\mathfrak{R} + \lambda_1\sqrt{-g} + \lambda_2\sqrt{-g}\,\gamma_{\mu\nu}w^{\mu\nu} + \sqrt{-\gamma}\,V(\gamma_{\mu\nu}w'^{\mu\nu}), \tag{5}$$

where $V(\gamma_{\mu\nu}w'^{\mu\nu})$ is an arbitrary function, and we have not explicitly show the dependence of Riemannian metric on the tensor and scalar parts. However, we exhibit the Minkowskian metric as well as the contractions with $w'^{\mu\nu}$ and $w^{\mu\nu}$. Of course, we have to add some terms with the Lagrange multipliers:

$$\mathfrak{L}_{\text{mult}} = \mathfrak{K}^{[1]}_{\mu\nu}\cdot[a^{\mu\nu} - \gamma^{\mu\nu}] + \mathfrak{K}^{[2]}_{\mu\nu}\cdot\left[w'^{\mu\nu} - \frac{1}{3}\left(\gamma_{\mu\nu} - \frac{D_\mu D_\nu}{D^2}\right)\omega'\right],$$

where we have put $w^{\mu\nu} = w'^{\mu\nu} + a^{\mu\nu}$. The multipliers allow us to take correct variations of the Lagrangian, while they are not very essential for the further consideration, since we put the tensor densities $\mathfrak{K}^{[n]}_{\mu\nu}$ equal to zero.

The physical meaning of terms added to the scalar curvature is rather transparent: we have introduced the variation of energy under the external source of gravitational field with respect to the flat vacuum. The term with a factor $\lambda_1$ is the usual cosmological constant in the General Relativity, and we have to introduce it in accordance with the astronomical observations. The second term with a factor $\lambda_2$ is chosen in the minimal model of linear dependence on the contraction of Riemannian and Minkowskian metrics. We will show that in the minimal model, the ratio $g/\gamma = \det g_{\mu\nu}/\det\gamma_{\mu\nu}$ will fix the scalar field. This suggestion on the form of third term significantly restricts the Lagrangian under study. We will mention on a possible generalization below, but concentrate the consideration on the minimal model as clarified above.

The minimal term with the factor $\lambda_2$ is necessary in order to satisfy the first principle of vacuum stability. The term with the potential $V$ is arbitrary to the moment, but we will present some arguments towards its fixing.

[3] At the global $\omega$, we have to define the action of singular operator $D^\mu D^\nu/D^2$. So, contracting $\gamma^{\mu\nu} = (\gamma^{\mu\nu} - D^\mu D^\nu/D^2)\,\omega_0/3$ with $\gamma_{\mu\nu}$, we find $\omega_0 = 4$, that implies the introduction of definition by the substitution $(D^\mu D^\nu/D^2)\,\omega \to \gamma^{\mu\nu}\omega/4$ at the global $\omega$.

[4] We accept the units, in which $4\pi G = 1/m^2_{\rm PL} = 1$, $G$ is the gravitational constant, $m_{\rm PL}$ is the Planck mass.

Therefore, the vacuum energy can shift under the source. This phenomenon is quite ordinary in the field theory, since, for instance, the vacuum energy of free massive scalar field $\varphi$ is shifted under a source $j$, so that the potential is given by

$$U(\varphi, j) = m_\varphi^2 \frac{\varphi^2}{2} - j\varphi = m_\varphi^2 \frac{(\varphi - j/m_\varphi^2)^2}{2} - \frac{j^2}{2m_\varphi^2},$$

and the density of vacuum energy is misplaced from zero at $j \neq 0$:

$$U_{\rm vac}(\varphi_{\rm vac}, j) = -\frac{j^2}{2m_\varphi^2},$$

and it depends on the source.

In MRTG the shift of vacuum energy under the source of Riemannian metric is introduced in contrast to the General Relativity, where the cosmological term is constant. The dependence of vacuum energy appears due to the graviscalar field, and we deal with the scalar-tensor extension of General Relativity.

Then we generally get the equations of motion

$$\frac{\delta \mathfrak{L}}{\delta \mathfrak{u}^{\mu\nu}} = \frac{\delta \mathfrak{L}}{\delta \mathfrak{g}^{\mu\nu}} - \frac{1}{3}\left(\gamma_{\mu\nu} - \frac{D_\mu D_\nu}{D^2}\right)\left(\gamma^{\alpha\beta}\frac{\delta \mathfrak{L}}{\delta \mathfrak{g}^{\alpha\beta}}\right), \tag{6}$$

$$\begin{aligned}\frac{\delta \mathfrak{L}}{\delta \omega'} &= \sqrt{-\gamma}\,\frac{1}{3}\left(\gamma^{\mu\nu} - \frac{D^\mu D^\nu}{D^2}\right)\frac{\delta \mathfrak{L}}{\delta \mathfrak{w}'^{\mu\nu}} \\ &= \sqrt{-\gamma}\,\frac{1}{3}\left(\gamma^{\mu\nu} - \frac{D^\mu D^\nu}{D^2}\right)\left(\frac{\delta \mathfrak{L}}{\delta \mathfrak{g}^{\mu\nu}} + \frac{\delta^\star \mathfrak{L}}{\delta \mathfrak{w}'^{\mu\nu}}\right),\end{aligned} \tag{7}$$

where the action of $\delta^\star$ implies the variation of terms explicitly depending on $\mathfrak{w}'^{\mu\nu}$ or $\mathfrak{w}^{\mu\nu}$.

In order to match with the General Relativity, we postulate that the form of field equations for the Riemannian metric is independent of the spin contents of $g_{\mu\nu}$. This uniform principle of MRTG reads off

$$\frac{\delta \mathfrak{L}}{\delta \mathfrak{g}^{\mu\nu}} = 0. \tag{8}$$

Then the field equations of motion for $\mathfrak{g}^{\mu\nu}$ have got the form

$$\frac{\delta \mathfrak{L}}{\delta \mathfrak{g}^{\mu\nu}} = -\frac{1}{4}\,R_{\mu\nu} + \frac{1}{2}\,\lambda_1\, g_{\mu\nu} + \frac{1}{2}\,\lambda_2\, g_{\mu\nu}\,\gamma_{\alpha\beta} w^{\alpha\beta} = 0, \tag{9}$$

According to the RTG principle of vacuum stability, in the limit of $u^{\mu\nu} \to 0$, $w^{\mu\nu} \to \gamma^{\mu\nu}$ the purely gravitational equations of motion with no matter have to be identically satisfied, so that we get

$$\lambda_1 = -4\lambda_2. \tag{10}$$

The graviscalar field equation of motion, taking into account the uniform principle, gives the expression

$$\frac{\delta \mathfrak{L}}{\delta \omega'} = \lambda_2\sqrt{-g} + \sqrt{-\gamma}\, V'(\omega') = 0, \tag{11}$$

so that in the limit of flat vacuum we find

$$V'(\omega')\big|_{\omega'=0} = -\lambda_2. \tag{12}$$

Let us emphasize that the uniform principle of equations for the gravitational field (8) combined with (6) and (7) implies that the Riemannian metric $g_{\mu\nu}$ and the graviscalar $\omega$ can be considered as independent, since their variations are nor related with each other and enter in the separate field equations

$$\begin{aligned} \frac{\delta \mathfrak{L}}{\delta \mathfrak{g}^{\mu\nu}} &= 0, && (13)\\ \frac{\delta^\star \mathfrak{L}}{\delta \mathfrak{w}'^{\mu\nu}} &= 0. && (14) \end{aligned}$$

This point is very important, because we have started with the graviscalar given by the contraction of Riemannian and Minkowskian metrics, but the further consideration has shown that the dependence of vacuum energy on the external gravitational source appears due to the connection of graviscalar with the ratio of Riemannian and Minkowskian metrics as follows from (11). We have found that the modification of RTG leads to the so-called "slightly bi-metric" theory, where the flat metric enters in the full Lagrangian by its determinant, only.

After taking into account the field equations, the density of energy-momentum tensor for the gravitational field can be written down in the form

$$\begin{aligned} \mathfrak{t}_g^{\mu\nu} = -2\frac{\delta \mathfrak{L}}{\delta \gamma_{\mu\nu}} &= -\frac{1}{4}\,\mathfrak{J}^{\mu\nu} - 2\,\lambda_2\,\sqrt{-g}\,w^{\mu\nu} \\ &\quad -\sqrt{-\gamma}\,V(\gamma_{\alpha\beta}w'^{\alpha\beta})\,\gamma^{\mu\nu} - 2\sqrt{-\gamma}\,V'(\gamma_{\alpha\beta}w'^{\alpha\beta})\,w'^{\mu\nu}, \end{aligned} \tag{15}$$

where the density of current [4] is equal to

$$\mathfrak{J}^{\mu\nu} = D_\alpha D_\beta(\gamma^{\alpha\mu}\mathfrak{g}^{\beta\nu} + \gamma^{\alpha\nu}\mathfrak{g}^{\beta\mu} - \gamma^{\alpha\beta}\mathfrak{g}^{\mu\nu} - \gamma^{\mu\nu}\mathfrak{g}^{\alpha\beta}).$$

In the absence of gravitational field, the vacuum solution has to lead to zero of introduced energy-momentum tensor, hence, we find

$$V(\omega')\big|_{\omega'=0} = -2\lambda_2. \tag{16}$$

The graviscalar field equation of motion (11) implicates a stringent restriction to the potential

$$\lambda_2\,\sqrt{-g} = -\sqrt{-\gamma}\,V'(\omega'), \tag{17}$$

which use provides the following form of energy-momentum tensor:

$$\mathfrak{t}_g^{\mu\nu} = -\frac{1}{4}\,\mathfrak{J}^{\mu\nu} + \sqrt{-\gamma}\,\gamma^{\mu\nu}\,[2V'(\omega') - V(\omega')]. \tag{18}$$

Then the covariant conservation law is valid, if we put

$$\left.\begin{aligned} D_\mu\,\mathfrak{g}^{\mu\nu} &= 0, \\ 2V'(\omega') &= V(\omega'), \end{aligned}\right\} \quad \Longrightarrow \quad D_\mu\,\mathfrak{t}_g^{\mu\nu} = 0. \tag{19}$$

The above conditions are sufficient, but necessary. Nevertheless, let us consider the density of graviton energy-momentum tensor at $\mathfrak{g}^{\mu\nu} = \mathfrak{w}^{\mu\nu}$, i.e. at zero tensor component. Then the covariant derivative

$$D_\mu\,\mathfrak{t}_g^{\mu\nu} = \sqrt{-\gamma}\,\gamma^{\mu\nu}\,D_\mu\,[2V'(\omega') - V(\omega')] = 0. \tag{20}$$

For arbitrary $\omega'$ we derive

$$2V'(\omega') = V(\omega') \tag{21}$$

under the normalization conditions in the flat space-time. However, the assumption on zero tensor contribution is quite synthetic, in general, since it can be inconsistent with the field equation. Therefore, in the modified RTG the potential of scalar field is determined under the clear motivation, which, however, does not provide a necessary condition for the conservation law.

Nevertheless, we can present an additional argument based on the minimal introduction of flat metric. In that case, one usually expresses the curvature with no involvement of covariant derivative with the connection consistent with the Minkowskian metric as ordinary given in the General Relativity. Then the variation of Lagrangian with respect to the flat metric, i.e. the density of gravitational energy-momentum tensor, does not include the term of $\mathfrak{J}^{\mu\nu}$, and eq. (20) is exact, which implies the strict derivation of the potential form $V$. Moreover, we emphasize that under both the General Relativity expression for the curvature and the potential condition (21), the energy-momentum tensor of gravitational field is independent of scalar potential and equal to zero

$$\mathfrak{t}_g^{\mu\nu}\Big|_{\rm GR} = 0,$$

which is ideologically correct in the purely geometrical theory.

Then in the minimal model of MRTG the motion equation for the scalar field can be exactly integrated out, so that

$$\begin{aligned} V(\omega') &= -2\lambda_2 \exp\left[\tfrac{1}{2}\omega'\right], \\ \omega' &= \ln\left[\frac{g}{\gamma}\right]. \end{aligned} \tag{22}$$

After the substitution, the transversity of tensor density $\mathfrak{t}_g^{\mu\nu}$ becomes explicit

$$\frac{1}{4}\, D^2 \mathfrak{g}^{\mu\nu} = \mathfrak{t}_g^{\mu\nu}, \tag{23}$$

if

$$D_\mu\, \mathfrak{g}^{\mu\nu} = 0.$$

whereas for the trace we obtain the expression

$$D^2\left(\gamma_{\mu\nu}\, \mathfrak{g}^{\mu\nu}\right) = 4\, \mathfrak{t}_g^{\mu\nu}\, \gamma_{\mu\nu}. \tag{24}$$

At small perturbations of gravitational field both the tensor and scalar components are massless, and the dimensional parameter determining the cosmological constant is not related to a nonzero mass of gravitons. So, we denote

$$\lambda_1 = \frac{m_\Lambda^2}{2}, \quad \lambda_2 = -\frac{m_\Lambda^2}{8},$$

while the masses are equal to zero:

$$m_u^2 = 0, \quad m_\omega^2 = 0.$$

This result could be expected, since the field equation of graviscalar allows us to express the determinant of Minkowskian metric in terms of both the determinant of Riemannian metric and the scalar field. This fact implies that in MRTG one can make the Minkowskian metric to enter the Lagrangian implicitly, and the only appearing of $\gamma$ is the determination of graviscalar value as well as the condition of transversity. This fact provides the breaking of strong equivalence principle by the inherent dependence on the value of graviscalar field related with the Minkowskian metric. We explain that this dependence is due to the shift of vacuum energy because of the external gravitational field, and this shift is the only point of such the breaking, while the weak equivalence of gravity and inertia is valid.

Let us make a note on the generalization of consideration by the substitution

$$\lambda_2\sqrt{-g}\,\gamma_{\mu\nu}w^{\mu\nu} \to \lambda_2\sqrt{-g}\,\tilde{V}(\gamma_{\mu\nu}w^{\mu\nu}),$$

where $\tilde{V}(4) = 4$ for the definiteness. In that case, the form of relation between $\lambda_1$ and $\lambda_2$ holds with no change, as well as the derivation concerning for the potential $V$ and the massless gravitational fields remains valid. The field equation of graviscalar reads off

$$\lambda_2\sqrt{-g}\,\tilde{V}'(\gamma_{\mu\nu}w^{\mu\nu}) + \sqrt{-\gamma}\,V'(\gamma_{\mu\nu}w'^{\mu\nu}) = 0. \tag{25}$$

Therefore, if the potential $\tilde{V}$ is arbitrary, the relation between the graviscalar and the ratio of metric determinants is indefinite. Moreover, some choices of $\tilde{V}$ could be inconsistent with the existence of meaningful solution of (25). Nevertheless, we can argue for the linear dependence of $\tilde{V}$. Indeed, the quantum loop corrections to the gravitational Lagrangian due to the matter fields result in the scale dependence of cosmological constant (see, for instance, [10]). So, the scalar field with a mass $m$ gives the following contribution:

$$m_\Lambda^2\big|_{\rm eff} = m_\Lambda^2\big|_{\rm bare} - \frac{m^4}{4\pi^2 m_{\rm PL}^2}\,\ln\frac{\mu_0}{\mu}, \tag{26}$$

The scale factor of $\mu_0/\mu$ can be evidently replaced by the ratio of curved and flat metrics:

$$\ln\frac{\mu_0}{\mu} = \frac{1}{8}\,\ln\frac{g}{\gamma} + \text{const}.$$

where we have explored a simple relation between the differentials

$$\mathrm{d}\ln\mu = -\mathrm{d}\ln\Omega,$$

where $\Omega$ is a dilution factor resulting in the scaling of metrics $g_{\mu\nu} = \Omega^2\,\gamma_{\mu\nu}$ in the case of conformal relation. The arbitrary constant in the above relation of scale factors can be fixed by the principle of vacuum stability, since we require that the vacuum is flat, and hence,

$$m_\Lambda^2\big|_{\rm eff} = -\frac{m^4}{32\pi^2 m_{\rm PL}^2}\,\ln\frac{g}{\gamma}.$$

Thus, the cosmological term should tend to zero in the vacuum state, while its dependence on the scale factor expressed in terms of the determinants is given by the renormalization procedure, i.e. the corresponding function is the log. Of course, this dependence can be modified by higher orders, a summation of perturbative contributions by various matter fields,

which can change the sign of factor in front of the logarithm. Nevertheless, we stress that in the leading order one could use the logarithmic variation of the cosmological term, that fixes the form of $\tilde{V}$ as the linear function. This fact reminds of the minimal prescription in the theory under study.

We stress that the above procedure giving the dependence of the cosmological term on the external gravitational source can be involved in the modified RTG, but the General Relativity, since this action breaks the strong equivalence principle in what concerns for the vacuum, which can enter the low-energy effective action.

Then the density of Lagrangian for the gravitational field can be represented in the form

$$\begin{aligned} \mathfrak{L} &= -\frac{1}{4}\mathfrak{R} + \sqrt{-\gamma}\,[4V'(\omega') - V'(\omega')\,\omega + V(\omega')] = \\ &= -\frac{1}{4}\mathfrak{R} + \frac{m_\Lambda^2}{8}\sqrt{-\gamma}\,(2-\omega')\exp\left[\frac{1}{2}\omega'\right]. \end{aligned} \tag{27}$$

The effective potential of scalar field in the flat Minkowskian space-time is equal to

$$V_{\text{eff}} = -\frac{m_\Lambda^2}{8}\sqrt{-\gamma}\,(2-\omega')\exp\left[\frac{1}{2}\omega'\right] = \frac{m_\Lambda^2}{8}\sqrt{-\gamma}\,U(\omega'), \tag{28}$$

where the dimensionless function $U(\omega')$ has got the only minimum corresponding to the flat vacuum of Riemannian space-time (see Fig. 2).

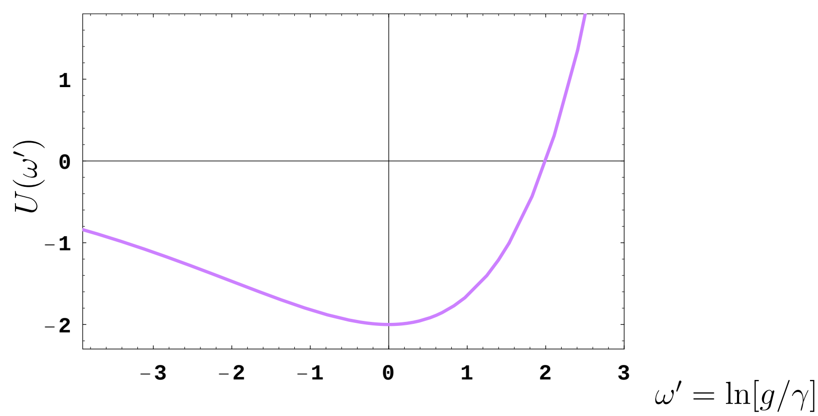


Figure 2: The dependence of scalar field potential in dimensionless units as follows from (28).

In terms of Riemannian metric, the potential can be transformed to the form

$$V_{\text{eff}} = -\frac{m_\Lambda^2}{8}\sqrt{-g}\,\left(2 - \ln\left[\frac{g}{\gamma}\right]\right), \tag{29}$$

where the vacuum Minkowskian metric enters explicitly.

It is evident that the geometrization principle for the interaction of gravitation with the matter provides the validity of above conclusions on the form of gravitational Lagrangian.

The motion equations are written down as

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R - \frac{m_\Lambda^2}{4} \ln\left[\frac{g}{\gamma}\right]\, g_{\mu\nu} = 2\, T^M_{\mu\nu}, \tag{30}$$

where the energy-momentum tensor of matter is defined under its Lagrangian density $\mathfrak{L}^M$ by the expression

$$T^M_{\mu\nu} = 2\, \frac{\delta \mathfrak{L}^M}{\sqrt{-g}\delta g^{\mu\nu}}.$$

Thus, we have completely determined the dynamics of gravitational field in RTG with the cosmological constant.

Finally, we give the formula for the Lagrangian density of gravitational field in the modified RTG

$$\mathfrak{L}_{\rm MRTG} = -\frac{1}{4}\mathfrak{R} - \frac{m_\Lambda^2}{8}\, \sqrt{-g}\, \omega' + \mathfrak{K}\cdot \left[\omega' - \ln\frac{g}{\gamma}\right], \tag{31}$$

where the external graviscalar field is fixed after the variation over the Lagrange multiplier $\mathfrak{K}$. The action with (31) results in the field equations (30) under the summation with the action of matter. In addition, if one follows the ideology of General Relativity with the minimal involvement of flat vacuum metric, the harmonic condition

$$D_\mu\, \mathfrak{g}^{\mu\nu} = 0,$$

is treated as the gauge condition fixing the arbitrary connection between the flat and Riemannian metrics, while in MRTG (nonzero $\mathfrak{J}^{\mu\nu}$) this is the consequence of conservation law (the second principle).

Two comments are to the point. First, we emphasize the different role of mass parameter in the modified and original versions of RTG. Certainly, in the theory under consideration the tensor graviton field is massless as it is in the General Relativity, while in the Logunov's RTG it is massive as well as the dynamical scalar field. In the modified RTG, the dimensional parameter $m_\Lambda$ can have a very different dynamical nature, since it is connected to the contribution of gravitational degrees of freedom into the energy-momentum tensor. So, in cosmological models the scale parameter can depend on both properties of physical vacuum in a full theory and interactions of gravitons with the matter. Another example could be considered in problems on a gravitational field with a point-like or rotating sources, where the scale parameter can depend on the position with respect to the source point or its axis. Moreover, the position dependence of $m_\Lambda$ could be adjusted to give some reasonable limits for such problems. Thus, we have the different treatment of scale parameter in the original RTG and its modified version under study. We stress ones more that the scale parameter in the modified RTG is related with the cosmological constant, however, it is not straightforwardly associated with the graviton or graviscalar mass, since the tensor graviton is massless, while the scalar field equation is algebraic, and hence, it is not dynamical.

The second comment concerns for the bimetric feature. Following authors of [11], we emphasize that the so-called "slightly bimetric" theory involving the determinant of Minkowskian metric into the Lagrangian only, is equivalent to the scalar-tensor gravity. Those bimetric theories in [11] are constructed under the same or very close principles accepted in RTG. So, the universal coupling of gravity, conservation law and presence of Minkowskian metric are declared. Then, an additional free parameter of Lagrangian is a flat cosmological term with

the Minkowskian metric. The gauge principle is explored to show that the energy-momentum tensor is fixed if only the gauge condition is chosen. In the slightly bimetric theories the gauge invariance is restricted, indeed. Such constructions are equivalent to general covariant theories plus a scalar field, so that every slightly bimetric theory has a scalar-tensor "twin" and *vice versa*. Thus, we do not see any constructive problem in the formulation of such bimetric theory. Another problem is an interpretation of bimetric theories since we deal with two null cones associated with the propagation of interactions. Indeed, for the particle physicists one considers the graviton as the interaction carrier with respect to the flat vacuum, i.e. the Minkowskian metric. Therefore, the principle of causality states that the propagation should be inside the null cone of this flat metric, which implies that the null cone of curved Riemannian metric should be inside the timelike region of Minkowskian one. However, this causality principle proceeds the inequality condition depending on the gauge change of Riemannian metric. One could to restrict the gauge arbitrariness to get some conclusions. Nevertheless, the problem reveals to be more complex. It is connected to the general interpretation for the case of intersecting null cones. The first opinion is supported by A.A.Logunov, who declared that all of solutions with the intersection of null cones are not physical, since only the causal propagation is permitted. The objections to that point of view are based on both the restricted gauge invariance arbitrary changing the sign in the causality inequality and the presentation of special solutions, which are physically reasonable, but demonstrate the intersections of null cones. For example, solutions for the gravitational field of massive rotating bodies far from the source (the so-called Lense-Tirring metrics) with the angular momentum large enough result in that the inclination (in the asimutal direction) of null cone for the Riemannian metric with respect to the null cone of Minkowskian metric can become so large even at large distances from the body that these cones intersect each other. Another opinion insists that the flat metric is a fiction because of the null cone problems. Both versions of interpretation suffer from the gauge-noninvariant formulation of arguments. Related questions are extensively discussed in [11] provided by a complete list of references. We can add the third point of view: the problem of causality in terms of null cones could be fictitious. Indeed, what one can observe is the null cones for the motion in the Riemannian metric, while the Minkowskian null cones are the almost-vacuum limits of Riemannian ones, so that the only quantity accessible for the observation in the slightly bimetric theory as MRTG (with the ratio of determinants for the flat and curved metrics) is the compression or dilution factor for the Riemannian space-time with respect to the Minkowskian one, if we suggest the gauge invariance of this factor. Anyway, in what follows, we deal with the Friedmann–Robertson–Walker metric, which is conformally flat, and therefore, the characteristic surfaces, i.e. the gravity and matter null cones coincide for this particular case. We will show the importance of cosmological term in this situation, while in other applications the value of scale parameter, in fact, can be put to zero due to some classical motivations like the minimization of energy or absence of hot gravitational field and so on.

Finally, let us make a note on the covariant conservation of energy-momentum tensor. In the General Relativity one has

$$\nabla_\mu g^{\mu\lambda} T^M_{\lambda\nu} = 0. \tag{32}$$

The physical meaning of (32) is the following: at arbitrary external gravitational source $g_{\mu\nu}$ interacting with the matter, the invariance of matter action with respect to the general coordinate transformations under the field equations leads to the conservation law. The introduction of vacuum energy, i.e. the cosmological constant, in the General relativity results in the con-

servation of

$$\tilde{T}^M_{\lambda\nu} = T^M_{\lambda\nu} + \Lambda g_{\lambda\nu}.$$

However, under the connection consistent with the Riemannian metric

$$\nabla_\mu g_{\lambda\nu} = 0,$$

eq. (32) remains valid since the energy-momentum tensors of both the matter and the vacuum are conserved separately.

In MRTG the full tensor $\hat{T}^M_{\mu\nu}$ of matter and vacuum is conserved, so that

$$\nabla_\mu g^{\mu\lambda} \hat{T}^M_{\lambda\nu} = \nabla_\mu g^{\mu\lambda} \left( T^M_{\lambda\nu} + \frac{m_\Lambda^2}{8} \ln\left[\frac{g}{\gamma}\right] g_{\lambda\nu} \right) = 0. \tag{33}$$

The physical meaning of (33) is certainly clear: under the introduction of external source in the form of gravitational field, the density of vacuum energy changes, i.e. the value of cosmological term depends on the Riemannian metric. Therefore, considering the action of matter in the external gravitational field, one has to take into account the vacuum contribution depending on the Riemannian metric, so that the variation of full action for the matter and the vacuum under the general coordinate transformations is equal to zero if (33).

## III. Cosmological solutions

In this section we analyze the scenario for the universe evolution in MRTG. From the very beginning it is clear that in MRTG due to the principle of vacuum stability the empty flat space-time is the solution of motion equations, and this fact will be illustrated in this section, so that the expansion of universe containing a matter takes place under an introduction of some instability. We will show that such the instability appears if the energy densities of the matter and cosmological term are approximately equal to each other, and the universe is compressed with respect to the flat vacuum. Then the expansion takes place if we consider the Lagrangian density with the negative square of mass parameter introduced above. Thus, the starting point of our analysis is the gravitation Lagrangian density written down in the form

$$\mathfrak{L} = -\frac{1}{4}\mathfrak{R} + \frac{\bar{\mu}_\Lambda^2}{8}\sqrt{-g}\,\omega' + \mathfrak{K}\cdot\left[\omega' - \ln\frac{g}{\gamma}\right]. \tag{34}$$

Consider the Riemannian metric, which gives the following Friedmann–Robertson–Walker interval:

$$ds^2 = dt^2 - a^2(t)\,[dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)]. \tag{35}$$

The proper-time presentation of Riemannian metric in (35) is consistent with the field equations of RTG (*viz.*, $D_\mu \mathfrak{g}^{\mu\nu} = 0$) [4], if the Minkowskian metric has the form

$$ds_\gamma^2 = \frac{1}{a^6(t)}\,dt^2 - \frac{1}{\varkappa^2}[dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)], \tag{36}$$

where $\varkappa$ is a global scale factor.

The cosmological term can be rewritten as the contribution into the energy-momentum tensor

$$T^\Lambda_{\mu\nu} = 2\,\frac{\delta}{\sqrt{-g}\delta g^{\mu\nu}}\left\{\frac{\bar{\mu}_\Lambda^2}{8}\sqrt{-g}\;\omega'\right\}_{\omega'=\ln\frac{g}{\gamma}} = -\frac{\bar{\mu}_\Lambda^2}{8}\ln\left[\frac{g}{\gamma}\right]\,g_{\mu\nu}. \tag{37}$$

The equations of motion (30) with the metric under study are transformed to the form

$$
\begin{aligned}
\frac{\ddot{a}}{a} &= -\frac{1}{3}\,(\rho+3p), \\
H^2 &= \frac{2}{3}\,\rho,
\end{aligned}
\tag{38}
$$

where we have introduced the standard notation for the Hubble constant $H = \dot{a}/a$ and written the energy-momentum tensor in terms of energy density $\rho$ and pressure $p$. The covariant conservation of energy-momentum tensor leads to the liquid equation, which follows from (38)

$$
\dot{\rho}a^3 + 3(\rho+p)a^2\dot{a} = 0. \tag{39}
$$

For the given metrics (35) and (36) we can easily find that $\ln g/\gamma = 12\ln a\sqrt{\varkappa}$. However, without a matter we expect that the cosmological constant is equal to zero, and we approach the stable vacuum solution, that implies $a^2(t)\,\varkappa \equiv 1$. Moreover, adjusting the limit of free vacuum $g_{\mu\nu} \to \gamma_{\mu\nu}$, we get the condition $\varkappa = 1$. Further, we introduce the notation $\mu_\Lambda^2 = 2\bar{\mu}_\Lambda^2$, which corresponds to the naive scale behaviour of $\ln g \sim 6\ln a$.

Let us study the case of dust matter with the cosmological term in MRTG. Then

$$
\rho_\Lambda = -\frac{3}{4}\,\mu_\Lambda^2\,\ln a, \quad p_\Lambda = -\rho_\Lambda, \tag{40}
$$

$$
\rho_M = \frac{3}{4}\,\mu_\Lambda^2\,\rho_0, \qquad p_M = 0, \tag{41}
$$

where we have introduced the dimensionless parameter $\rho_0$. Equations (38) lead to the expressions

$$
\dot{H} = -\rho_M, \tag{42}
$$

$$
H^2 = \frac{\mu_\Lambda^2}{2}\left[\ln\frac{1}{a} + \rho_0\right], \tag{43}
$$

and

$$
\dot{\rho}_0 = H\,(1-3\rho_0). \tag{44}
$$

Putting $\rho_0$ equal to its stable value[5] at late times we find that the matter density is constant, and

$$
\rho_0 = \frac{1}{3}, \quad \rho_M = \frac{\mu_\Lambda^2}{4}. \tag{45}
$$

Then the time-equations are easily integrated out, so that

$$
\begin{aligned}
a(t) &= \frac{1}{k}\,\exp\left[-\left(\sqrt{-\ln(a_0k)} - \frac{\mu_\Lambda}{2\sqrt{2}}t\right)^2\right], \\
H^2 &= \frac{\mu_\Lambda^2}{2}\left(\sqrt{-\ln(a_0k)} - \frac{\mu_\Lambda}{2\sqrt{2}}t\right)^2, \\
\dot{H} &= -\frac{\mu_\Lambda^2}{4},
\end{aligned}
\tag{46}
$$

[5] Here it is important that $H > 0$.

where $k = \exp[-\rho_0]$, and the initial data at zero time have to satisfy the condition $\ln(a_0 k) < 0$.

We see that the universe has got the exponential, inflationary expansion with the constant density of matter, which is significantly less than the density of cosmological term at early times of initial stage. The cosmological term with respect to the flat Minkowskian space-time is naturally interpreted as the contribution of hot strongly coupled gravitons. The density of gravitons falls with the time of inflation, while the entropy is transmitted from the gravitons to the matter, so that the matter entropy grows exponentially.

The deceleration parameter $q = -\frac{\ddot{a}}{a}\frac{1}{H^2}$ is determined by the relation

$$q = -1 - \frac{\dot{H}}{H^2}, \tag{47}$$

and at the initial stage it is close to $-1$, while in the end of inflation it becomes close to the value observed experimentally. So, for example, we get

$$q\big|_{(\rho_0 - \ln a) = 1} = -\frac{1}{2}, \quad q\big|_{(\rho_0 - \ln a) = 3/4} = -\frac{1}{3}.$$

The initial conditions of inflation for the flat universe in MRTG are determined at the Planck scale, so that we can naturally suggest that fluctuations are in the following range:

$$\begin{aligned} \delta a &\sim a, \quad a \to 0, \\ \delta t &\sim \frac{1}{m_{\rm PL}}, \end{aligned}$$

hence,

$$H_0^2 \sim m_{\rm PL}^2 \implies \ln\frac{1}{a_0} \sim \frac{m_{\rm PL}^2}{\mu_\Lambda^2}.$$

This fact implies that at low values of dimensional parameter in MRTG the inflation has got a huge scale, that is necessary for an achievement of homogeneous and isotropic thermal equilibrium before the transition to the regime of hot expanding universe (see [8]).

For the relativistic matter

$$\rho_{RM} = \frac{3}{4}\,\mu_\Lambda^2\,\rho_{0R}, \qquad p_{RM} = \frac{1}{3}\,\rho_{RM}, \tag{48}$$

we can write down analogous expressions, such as, for example,

$$\dot{\rho}_{0R} = H\,(1 - 4\rho_{0R}). \tag{49}$$

Choosing the stable value, we get

$$\rho_{0R} = \frac{1}{4}, \quad \dot{H} = -\frac{\mu_\Lambda^2}{4} = -\frac{4}{3}\,\rho_{RM}. \tag{50}$$

Other expressions for the solution remain valid after the corresponding redefinition of normalization parameter for the relativistic case, so that $k_R = \exp[-\rho_{0R}]$.

Let us stress that the given solutions for the cosmological evolution have been obtained with no reference to the statistical thermal properties of gravitons and matter. For example, let us consider the relations, which take place for the relativistic gas of non-interacting particles.

Their densities of energy $\rho_M$ and entropy $s_M$ versus the temperature $T$ are given by the expressions

$$\begin{aligned} \rho_M &= \frac{\pi^2}{30}\mathcal{N}T^4, \\ s_M &= \frac{2\pi^2}{45}\mathcal{N}T^3, \end{aligned} \tag{51}$$

where the number of degrees of freedom is equal to the sum over the polarization states of bosons and fermions with the masses less than the temperature, so that

$$\mathcal{N} = \mathcal{N}_B + \frac{7}{8}\mathcal{N}_F.$$

The presented cosmological solutions show that the ultra-relativistic ideal gas has the constant density of energy, and, hence, it has got a constant temperature. Thus, the inflationary expansion takes place isothermically for the matter, therefore its total entropy increases as $S_M = s_M\, a^3(t)\, V_0$. It is evident that this process can occur only under the loss of entropy by gravitons. The qualitative picture of universe inflation is the following: in the starting point of inflation at the Planck scale or so, the gravitons have got the large density in comparison with the dilute matter, so that the strong gravitational self-interaction of gravitons leads to a small scattering length (probably about the Planck length), and the gravitons interact with each other, while the matter density is small, and the transparent matter, in practice, evolves isothermically, since the scattering length of gravitons in the matter is much greater than the self-interaction length of gravitons. Therefore, in this approximation we can suppose that the matter, "in practice, weakly" interacts with the gravitational field. In the end of inflation the density o gravitons decreases to the values close to the parameters of matter, i.e. the universe little deviates from the flat one with the Minkowskian metric. Actually, the graviton scattering length off graviton becomes approximately equal to the scattering length off matter. At this point, the matter state equation differs from that of relativistic particles weakly interacting with the gravity. Probably, the warm gravitons heat up the matter, until the graviton density becomes too low to make a little influence on the cosmological evolution. We see that the universe inflation results in the dominant interaction of gravitons with the matter. Indeed, the scattering length is defined in terms of cross section $\sigma$ and particle density $n$

$$\lambda = \frac{1}{\sigma\, n},$$

therefore, if we naively estimate the particle density by the energy density divided by the temperature

$$n \approx n_0\,\frac{\rho}{T},$$

and put the cross section equal to

$$\sigma = \text{const}\,\frac{\alpha_{\text{GR}}^2}{T^2}\bigg|_{\alpha_{\text{GR}}=\frac{T^2}{m_{\text{PL}}^2}} = \text{const}\,\frac{T^2}{m_{\text{PL}}^4},$$

then we find the ratio of graviton scattering lengths off the matter and gravitons

$$\frac{\lambda_M}{\lambda_{\text{GR}}} = \frac{T_{\text{GR}}(t)\,\ln\frac{1}{a(t)}}{T_M\,\frac{1}{4}},$$

where the dependence of graviton temperature on the time $T_{\rm GR}(t)$ is determined by the state equation: density-temperature-volume, for the gravitons. So, we see that, if the temperature of gravitons is close to that of matter, then the tending of scale factor $a$ to unity during the inflation leads, initially, to the equalizing of scattering lengths, and further to the dominant interaction of gravitons with the matter.

If the total entropy of gravitational field and matter is conserved, then we have got the following expression for the entropy per a unit volume $V_0$ (in comoving coordinates independent of time, see the definition of interval):

$$S_0 = a^3(t)\, s_M(T) + S_{\rm GR}.$$

Further, we write down the standard relation for the total energy of gravitational field

$$d\mathcal{E}_{\rm GR} = T_{\rm GR}\, dS_{\rm GR} - p_\Lambda\, dV, \tag{52}$$

where, evidently,

$$\begin{aligned} V(t) &= a^3(t), \\ \mathcal{E}_{\rm GR} &= \rho_\Lambda(t)\, a^3(t), \\ p_\Lambda &= -\rho_\Lambda, \end{aligned} \tag{53}$$

so that after taking the derivative with respect to time we find

$$T_{\rm GR} = -\frac{V(t)\,\dot\rho_\Lambda}{s_M(T)\,\dot V(t)} = \frac{\mu_\Lambda^2\, m_{\rm PL}^2}{4 s_M(T)}.$$

Therefore, we have got that the temperature of gravitons also does not change during the time of inflation

$$T_{\rm GR} = \frac{45}{8\pi^2\mathcal{N}}\,\frac{\mu_\Lambda^2\, m_{\rm PL}^2}{T^3},$$

and the process is isothermic for the gravitons, too. Moreover, comparing the state equation of matter (51) with the stable density of matter in the inflation

$$\rho_M = \frac{3}{16}\,\mu_\Lambda^2\, m_{\rm PL}^2,$$

we find that

$$T_{\rm GR} = T,$$

i.e. the temperature of gravitons is equal to the temperature of matter during the inflation.

The equality of temperatures and the adiabaticy of process can be obtained also from the general equation of state (52) combined for the gravitons and matter, if we take into account (53) and analogous equations for the relativistic matter with the stable density. Indeed, calculating the differentials in the equation

$$d\mathcal{E}_{\rm GR} + d\mathcal{E}_M = (T_{\rm GR} - T_M)\, dS_{\rm GR} - p_\Lambda\, dV - p_M\, dV,$$

we get

$$(T_{\rm GR} - T_M)\, dS_{\rm GR} = 0,$$

so that, if the process is adiabatic for the closed system, then the temperatures of gravitons and matter are equal to each other.

Further, we have to emphasize two circumstances. First, at $H < 0$, i.e. after the end of inflationary isothermic expansion and the transition to the contraction, the point of constant density of matter $\rho_{0\mathrm{R}}$ becomes **unstable** in accordance with (49). Second, for the gauge non-gravitational interactions of matter with the coupling constant $\alpha$ we can estimate the cross section by

$$\sigma_{\mathrm{gauge}} = \sigma_0\,\frac{\alpha^2}{T^2},$$

so that the scattering length in such interactions of matter is significantly less than the graviton scattering length off matter,

$$\lambda_{\mathrm{gauge}} \sim \frac{1}{\alpha^2\,T} \ll \lambda_M \sim \frac{m_{\mathrm{PL}}^4}{T^5}, \tag{54}$$

This fact implies that in the beginning of inflation, when the graviton scattering off gravitons dominates, the isothermic adiabatic expansion takes place, while in the end of inflation, when the graviton scattering length off matter closes to the self-scattering length, the gravitons begin to lose the energy and heat up the matter, so that the absorbed energy of gravitons is transformed into the thermal motion due to the gauge interactions because of (54). Therefore, both the density and entropy of matter grow in the comoving coordinates. This increase is permitted, since the stability of constant value for the matter density is destroyed. By the process of such heating, at the time $t_1$ the energy density takes the form

$$\rho_{\mathrm{I}} = \frac{3}{4}\,\tilde{\mu}_{\mathrm{I}}^2\,m_{\mathrm{PL}}^2\left(\ln\frac{1}{a_1\,k_{\mathrm{R}}} + \frac{\Delta\rho_{\mathrm{IR}}}{a_1^4}\right), \tag{55}$$

$$\rho_{\mathrm{II}} = \frac{3}{4}\,\tilde{\mu}_{\mathrm{II}}^2\,m_{\mathrm{PL}}^2\left(\ln a_1\,k_{\mathrm{R}} + \frac{\Delta\rho_{\mathrm{IIR}}}{a_1^4}\right), \tag{56}$$

where $a_1 = a(t_1)$, and by (49) we put

$$\rho_{\mathrm{IR}}(t) = \frac{1}{4} + \frac{\Delta\rho_{\mathrm{IR}}}{a^4(t)}, \quad \rho_{\mathrm{IM}} = \frac{3}{4}\,\tilde{\mu}_{\mathrm{I}}^2\,m_{\mathrm{PL}}^2\left(\frac{1}{4} + \frac{\Delta\rho_{\mathrm{IR}}}{a^4}\right), \tag{57}$$

$$\rho_{\mathrm{IIR}}(t) = \frac{1}{4} - \frac{\Delta\rho_{\mathrm{IIR}}}{a^4(t)}, \quad \rho_{\mathrm{IIM}} = \frac{3}{4}\,\tilde{\mu}_{\mathrm{II}}^2\,m_{\mathrm{PL}}^2\left(-\frac{1}{4} + \frac{\Delta\rho_{\mathrm{IIR}}}{a^4}\right), \tag{58}$$

where $\Delta\rho_{\mathrm{R}}$ is the constant of integration, which we define in accordance with two variants of process development, since the parameter $\tilde{\mu}^2$ modified due to the graviton absorption can take two signs: in the second variant this parameter changes the sign in comparison with its state before the absorption $\mu_\Lambda^2$, while in the first variant the sign remains with no change. The physical sense of these two variants will be shown below under the description of their cosmological differences.

The nature of $\mu^2$ fall off may be twofold. First, we stress the essential fact of instability for the matter density, so that the absorption of gravitons can play a role of stabilization, and under the change of sign $\mu_\Lambda^2 \to -\tilde{\mu}_{\mathrm{II}}^2$ the stability of potential is restored. Second, it may be important that the changing scale of evolution $a(t)$ corresponds to an introduction of renormalization group scale dependence of matter charges and Yukawa-like couplings, so that the physical phase state rearrangement is possible, since such critical parameters as the temperature of phase transition, in general, depend on the mentioned running coupling constants.

A powerful absorption of gravitons leads to

$$\left|\ln \frac{1}{a_1\, k_{0\mathrm{R}}}\right| \ll \frac{\Delta\rho_{\mathrm{R}}}{a_1^4},$$

and we get the evolution equation

$$H^2 = \frac{1}{2}\,\tilde{\mu}^2_{\mathrm{I,II}}\, m^2_{\mathrm{PL}}\, \frac{\Delta\rho_{\mathrm{I,IIR}}}{a^4(t)}, \tag{59}$$

that implies the transition to the scenario of hot expanding universe, because

$$a^2(t) = a_1^2 + \sqrt{2}\,\tilde{\mu}_{\mathrm{I,II}}\, m_{\mathrm{PL}}\,(t - t_1), \tag{60}$$

and at late times for the relativistic matter we obtain

$$a(t) \sim \sqrt{t}.$$

The temperature of hot universe $T_{hot}$ is determined from the comparison of energy density in the thermal equilibrium with the expression given in terms of $\rho_{0\mathrm{R}}$, so that

$$\frac{\pi^2}{30}\,\mathcal{N}\, T^4_{hot} \approx \frac{3}{4}\,\tilde{\mu}^2_{\mathrm{I,II}}\, m^2_{\mathrm{PL}}\, \frac{\Delta\rho_{\mathrm{R}}}{a_1^4},$$

wherefrom we have got an ordinary relation

$$T(t) \sim \frac{1}{a(t)}.$$

Since the absorption of gravitons by the matter takes place at the constant volume and with no loss of energy, we have introduce the notation $\tilde{\mu}$ for the dimensional parameter after the absorption, so that, evidently, comparing the total energy before the absorption of gravitons and after it, we get

$$\tilde{\mu}^2_{\mathrm{I}} \approx \tilde{\mu}^2_{\mathrm{II}} \approx \mu^2_{\Lambda}\, \frac{\ln \frac{1}{a_1\, k_{\mathrm{R}}}}{\frac{\Delta\rho_{\mathrm{R}}}{a_1^4}} \quad\Rightarrow\quad \tilde{\mu}^2 \ll \tilde{\mu}^2_{\Lambda}.$$

Thus, the stage of hot expanding universe can be quite long in time in order to have no contradiction with the astronomical observations.

The hierarchy of scales $\mu_\Lambda$ and $\tilde{\mu}$ implies also that at the graviton absorption

$$\mu^2_{\Lambda}\,\frac{1}{4} \approx \tilde{\mu}^2\, \frac{\Delta\rho_{\mathrm{R}}}{a_1^4},$$

and the heating up the matter is little in comparison with the stage of inflation.

Further, the given consideration can be analogously performed for the dust matter having a zero pressure. Then we get

$$\rho_0(t) = \frac{1}{3} \pm \frac{\Delta\rho_0}{a^3(t)},$$

and the law of hot universe expansion is ordinary again

$$a(t) \sim \sqrt[3]{t^2}.$$

Let us also consider the problem on the reaching the "critical" zero density of matter in the second scenario of postinflationary evolution. Indeed, at $a(t) > \sqrt[4]{4\Delta\rho_{\rm IIR}}$ the density of matter formally becomes negative, that has no physical sense for nonzero modes, if we do not suppose that this situation is possible under, first, the vacuum energy is negative in the state equations of matter, and, hence, the null-point of energy measure is posed at a negative energy; second, it is necessary that $\ln a \gg 1$ and a correct cosmological solution could take place: $H^2 = \frac{2}{3}\rho > 0$, though, as clear from the astronomical observations, this condition is valid.

The notion on the connection of matter density with the nonzero value of effective potential in the state of matter vacuum is important, because we see that at the inflation the stable isothermic density of matter energy $V_M^{inf}(0) \sim \mu_\Lambda^2 m_{\rm PL}^2 > 0$, while in the hot universe the matter density tends to a stable value of $V_{\rm IM}(0) > 0$ or $V_{\rm IIM}(0) < 0$ depending on the variant of development.

Finally, we note also that the one-loop quantum corrections lead to a renormalization of cosmological constant [10], so that in MRTG they determine the adiabatic dependence of dimensional parameter $\mu_\Lambda$ on the scale $a$, since it can be related with the ultraviolet cut off $M$ by the formula $d\ln M = -d\ln a$. In this work we do not discuss a motivation and numerical estimates of dimensional parameters for the cosmological evolution in MRTG.

A short remark should be done on a value of matter density fluctuations, which determine the large scale structure of Universe. It is clear that, if the graviton scattering length off matter is close to the scattering length in the gauge interactions of matter, then the fluctuations are about $\delta\rho/\rho \sim 1$. At the moment of inflation transition to the regime of hot universe we expect that

$$\frac{\delta\rho}{\rho} \approx \frac{\lambda_{\rm gauge}}{\lambda_M} \sim \frac{1}{\alpha^2}\left(\frac{c_{\mathcal N} T}{m_{\rm PL}}\right)^4.$$

The astronomical observations agree with the altitude of initial fluctuations

$$\frac{\delta\rho}{\rho} \sim 10^{-(4-5)},$$

that leads to

$$\frac{T}{m_{\rm PL}} \sim 10^{-(2-4)},$$

if $\alpha \sim 10^{-2}$ and the numerical factor $c_{\mathcal N}$ in front of temperature can be about $\mathcal{N} \sim 1 - 10^2$.

Thus, we have analyzed the cosmological solutions in the modified RTG and found the conditions of inflationary expansion as well as transition to the ordinary evolution of hot universe.

## IV. Conclusion

In the present paper we have shown that the cosmological scenario of universe evolution possesses some advantages in the modified Relativistic Theory of Gravitation, which naturally introduces the dependence of gravitational field action on the vacuum metric of Minkowski at nonzero cosmological term necessary in accordance with the astronomical observations. Certainly, the evolution equations allow the development of logically sound stages providing the inflationary expansion and the modern epoch of late hot Universe. In this way the inflation

takes place isothermically, and we have not to introduce a mechanism for a forced heating up a overcooled homogeneous and isotropic universe under the transition from the inflation to the hot universe. In MRTG the following stages of cosmological evolution are theoretically sound:

1. The beginning: a hot state with a Planck-scale density of both ultra-relativistic matter and gravitons; $\rho_M \approx \rho_\Lambda \sim m_{\text{PL}}^4$ with a temperature of $T_{\star\text{GR}} \sim T_{\star M} \sim m_{\text{PL}} \sim 10^{19}$ GeV in a strongly compressed state $\ln\frac{1}{a_\star} \gg 1$; a time $t_\star$.

2. The preinflation: a stage of expansion and matter cooling down a temperature $T_M$, so that $T_M^4 \sim \mu_\Lambda^2 m_{\text{PL}}^2$, where $\mu_\Lambda^2 \sim \frac{m_{\text{PL}}^2}{-\ln a_\star} \ll m_{\text{PL}}^2$; a scale of $T_M \sim 10^{14-16}$ GeV; the time changes from $t_\star$ to $t_0$, whereas $\ln\frac{1}{a(t_0)} \gg 1$.

3. The isothermic hot inflation with $T_{\text{GR}} = T_M$ up to a time $t_1$, when the density of gravitons falls off a level of matter density $|\ln a(t_1)| \sim 1$.

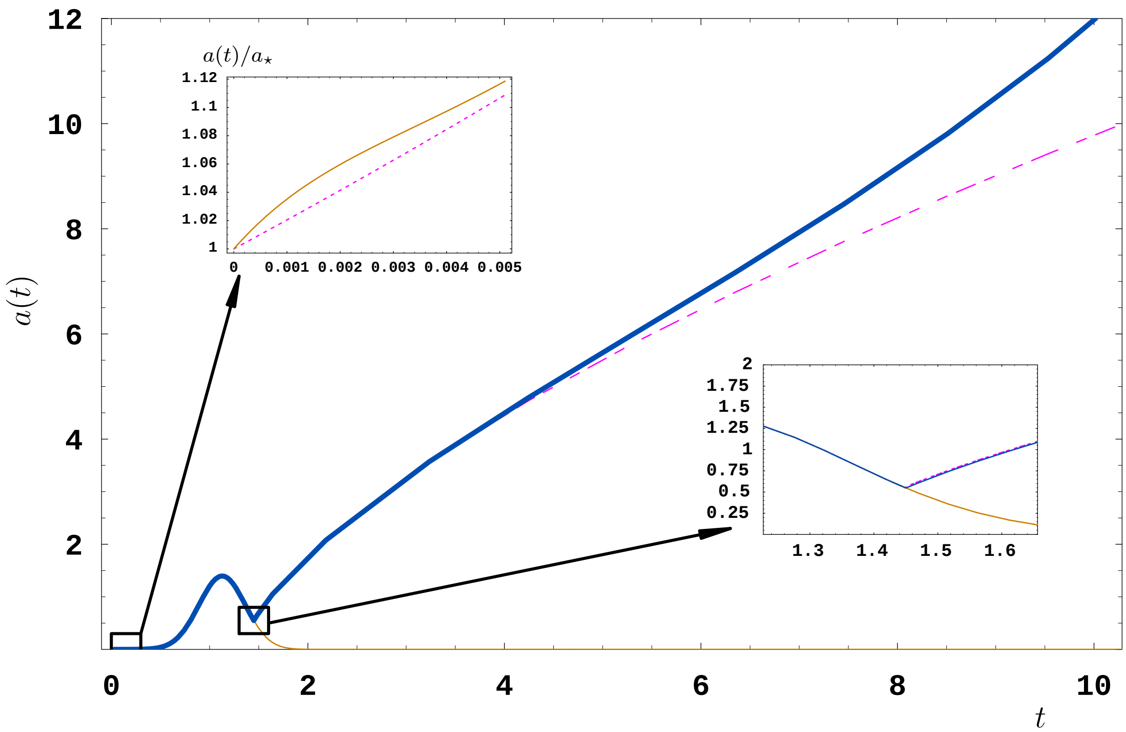


Figure 3: Stages of universe evolution in accordance with items 1-6. In the begin stage we show the scenario of cooling down below the Planck temperatures before the inflation in comparison with the pure inflation (the dotted line). The unstable regime change takes place under the graviton absorption to the process of hot universe expansion in comparison with the rapid exponential contraction. At late times we present the deviation from the regime of hot universe expansion (the dashed curve) due to a cold inflationary expansion (the solid curve). The time $t$ and scale $a(t)$ units are modelled and far away from the real ones.

4. The short stage of instability $H < 0$, when the energy of gravitons is converted to the thermal energy of matter due to the absorption of gravitons, so that before the absorption

$\rho_{\rm GR}(t_1) \sim \rho_M(t_1)$, while after it $\tilde\rho_{\rm GR} \ll \tilde\rho_M$; the restoration of stability[6] $H > 0$, whereas the matter is slightly heated up, and the evolution parameter $\tilde\mu^2/\mu_\Lambda^2 \sim \tilde\rho_{\rm GR}/\rho_{\rm GR} \ll 1$.

5. The present: the epoch of hot universe expansion with the matter temperature $T(t) \sim \frac{1}{a(t)}$ down to a temperature $T_c$, so that $T_c^4 \sim \tilde\mu^2 m_{\rm PL}^2$. Today we are "close" to the end of this stage because in the case of nonrelativistic cosmological motion of matter the deceleration parameter

$$q = -1 + \frac{3}{2}\frac{\rho_M}{\rho_M + \rho_{\rm GR}} \approx -0.33 \pm 0.17,$$

gives the witness that $\rho_{\rm GR} \sim (1-2)\rho_M$, i.e. the matter density is critically low.

6. The future II: a stable density of matter energy $V_M(0) < 0$, the infinite cold isothermic inflation with a logarithmically small density of matter. The speed of inflation here could be "damped" by a parametric adiabatic decrease of parameter $\tilde\mu^2$ down to zero because of a renormalization scale-dependent behaviour, that suggests the tending of matter vacuum density to zero, $V_M(0) \to 0$. This scenario of future with $V_M(0) < 0$ is the most probable in the light of astronomical data on the deceleration parameter, which is negative at the moment.

The future I: a stable density of matter energy $V_M(0) > 0$, the expansion of universe up to the equalizing of matter density with the negative contribution of cosmological term, which provides $q > \frac{3}{2}$, that contradicts the current data, the later stage of instability of cold universe $H < 0$, so that whether the restoration of stability will lead to an expansion again as in items 4-5, but for the overcooled universe with a further repetition of transition to a future according to item 6, or the contraction of universe will take place with significant fluctuations of density for the matter heated up.

Thus, in MRTG we have got quite the whole picture of universe evolution.

The author expresses his gratitude to prof. M.A.Mestvirishvili for fruitful discussions, prof. V.A.Petrov for an encouraging interest to the studies and thanks Dr. V.O.Soloviev for questions, remarks and suggestions.

This work is in part supported by the Russian Foundation for Basic Research, grants 01-02-99315, 01-02-16585 and 00-15-96645.

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[6]For the relativistic matter the evolution equations:

$$H^2 = \frac{2}{3}(\rho_M + \rho_{\rm GR}), \quad \frac{\ddot a}{a} = H^2 - \frac{4}{3}\rho_M,$$

result in the inflection point at $\rho_{\rm GR} = \rho_M$, while after the graviton absorption we get $\tilde\rho_M \gg \tilde\rho_{\rm GR}$, so that the transition from the concave regime of contraction to the convex curve of expansion is possible (see Fig. 3).

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